First order differential equation

Solve the following equation: $dy/dx = \dfrac{e^x - y}{x}$

Solution

Given:
$$\frac{dy}{dx} = \frac{e^x - y}{x}$$

Rewriting:
$$x\,dy + (y - e^x)\,dx = 0$$

We observe that $\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$, where $M = x$ and $N = y - e^x$. Integrating:

$$\int x\,dy = xy$$

$$\int (y - e^x)\,dx = yx - e^x$$

Therefore:
$$xy - e^x = C$$

Thus,
$$y(x) = \frac{e^x + C}{x}$$